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THE CONSTRUCTION OF AN INTEGRAL REPRESENTATION OF THE SOLUTION OF THE EQUILIBRIUM EQUATIONS OF TIMOSHENKO-TYPE THEORY FOR SHELLS OF COMPLEX GEOMETRY†

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A method of constructing an integral representation of the solution of the equilibrium equations of Timoshenko-type theory for thin or shallow isotropic shells of complex geometry is proposed. The method involves the following steps: writing the equilibrium equations for a fundamental solution of the three-dimensional theory of elasticity—the Kelvin vector in a curvilinear system of coordinates, normally to the middle surface of the shell; selecting a differential operator corresponding to the given theory of shells from the exact equilibrium equations for the Kelvin vector and constructing an integral representation of the vector of displacements of elements of the shell using Green's formula for the differential operator of the given theory of shells. It is shown that problems of determining the parameters of the stress–strain state of a shell in differential and integral formulations are equivalent, with an error which is small in the context of approximations of the theory. One method of constructing integral equations for the displacement vector of the elements of a shell of constant thickness is proposed. © 1998 Elsevier Science Ltd. All rights reserved.

An iteration algorithm for solving problems in the statics of shallow shells has been proposed [1–3] to overcome difficulties which arise when using the boundary-element method to determine the parameters of the stress–strain state of shells of complex geometry. However, it is noted in [3] that a large number of iterations is required if the radii of curvature of the shell are small under certain boundary conditions, and in some cases the process may diverge.

The method described below of obtaining an integral representation of the solution of the equations of equilibrium of the Timoshenko-type theory of shells does not have this drawback, is fairly universal, does not require the construction of a fundamental solution of the equilibrium equations, and allows the construction of second-order Fredholm integral equations for determining the unknown parameter on the contour and the middle surface of a shell of complex geometry.

1. In three-dimensional Euclidean space R^3, we are given an isotropic thin or shallow shell of constant thickness $2h$. The shell has a middle surface S which satisfies the necessary smoothness requirements, a piecewise-smooth contour of that surface Γ and a lateral surface Σ formed as the normal $\mathbf{m}$ moves towards S along Γ. We assume that the surface S is parametrized in terms of the radius vector $\mathbf{r}_x(x^1, x^2)$ (x^1, x^2 are curvilinear Gaussian coordinates). Then the radius vector of elements of the shell can be represented in the form $\mathbf{R} = \mathbf{r}_x + z\mathbf{m}(\mathbf{r}_x) = \mathbf{m}_x$, where $\mathbf{m}(\mathbf{r}_x) = \mathbf{m}_x$ is the unit normal to the surface S at a point with radius vector $\mathbf{r}_x$; $z \in [h, h]$ is the normal coordinate.

The three-dimensional equilibrium equation in vector form for elements of the shell has the form (everywhere below Greek subscripts and superscripts take values 1, 2, Latin subscripts and superscripts take values 1, 2, 3 and summation is performed over repeated indices)

$$\frac{1}{\sqrt{g}}[\partial_{\alpha,x}(\sqrt{g}\mathbf{P}^\alpha)+\partial_{3,x}(\sqrt{g}\mathbf{P}^3)]+\mathbf{F}=0 \quad \left(\partial_{\alpha,x}\equiv\frac{\partial}{\partial x^\alpha},\ \ \partial_{3,x}\equiv\frac{\partial}{\partial z}\right) \tag{1.1}$$

where $\mathbf{P}^i$ are the vector components of the stress tensor, $\mathbf{F}$ is the vector of mass forces and g is the determinant of the metric tensor.

In a system of coordinates associated with the middle surface of the shell, the vectors of the basis as well as the components of the metric tensor can be represented in the form (δ_α^β is the Kronecker delta, b_α^β are mixed components of the second metric tensor of the middle surface of the shell and α and β vary cyclically in determining the vectors of a mutual basis)

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$$\mathbf{R}_\alpha=(\delta_\alpha^\beta-zb_\alpha^\beta)\mathbf{r}_\beta,\quad \mathbf{R}^\alpha=[\mathbf{R}_\beta\times\mathbf{m}_x]/(\mathbf{R}_\alpha,\ [\mathbf{R}_\beta\times\mathbf{m}_x])$$
$$\mathbf{R}^3=\mathbf{R}_3=\mathbf{m}_x,\quad g_{\alpha\beta}=(\mathbf{R}_\alpha,\ \mathbf{R}_\beta),\quad g^{\alpha\beta}=(\mathbf{R}^\alpha,\ \mathbf{R}^\beta) \tag{1.2}$$

The vector components of the stress tensor can be expressed by the generalized Hooke's law in terms of the displacement vector of elements of the shell $\mathbf{u}$ by the formulae

$$\mathbf{P}^i=\lambda(\mathbf{R}^k,\ \partial_{k,x}\mathbf{u})\mathbf{R}^i+\mu(\mathbf{R}^i,\ \partial_{k,x}\mathbf{u})\mathbf{R}^k+\mu g^{ik}\partial_{k,x}\mathbf{u} \tag{1.3}$$

where λ and μ are Lamé parameters.

In the context of an approximate thin or shallow shell of constant thickness $2h$ the vectors of the basis, the components of the metric tensor, and also its determinant can, by (1.2), be replaced with error zb_α^β compared with δ_α^β by the corresponding quantities for the middle surface of the shell.

Using Eq. (1.1) and the usual approximations, we obtain equations for the vector of forces and vector of moments of the form

$$\partial_{\alpha,x}(\sqrt{a}\mathbf{T}^\alpha(\mathbf{r}_x))+\sqrt{a}\mathbf{X}(\mathbf{r}_x)=0 \tag{1.4}$$

$$\partial_{\alpha,x}(\sqrt{a}\mathbf{M}^\alpha(\mathbf{r}_x))+\sqrt{a}([\mathbf{r}_\alpha\times\mathbf{T}^\alpha(\mathbf{r}_x)]+\mathbf{Y}(\mathbf{r}_x))=0 \tag{1.5}$$

The vector of forces $\mathbf{T}^\alpha$ and of moments $\mathbf{M}^\alpha$ are given by the formulae

$$\mathbf{T}^\alpha(\mathbf{r}_x)=\int_{-h}^{h}\mathbf{P}^\alpha(\mathbf{r}_x,\ z)dz,\quad \mathbf{M}^\alpha(\mathbf{r}_x)=\left[\mathbf{m}(\mathbf{r}_x)\times\int_{-h}^{h}\mathbf{P}^\alpha(\mathbf{r}_x,\ z)zdz\right] \tag{1.6}$$

$$\mathbf{X}(\mathbf{r}_x)=\mathbf{P}_n^{(+)}+\mathbf{P}_n^{(-)}+\int_{-h}^{h}\mathbf{F}(\mathbf{r}_x,\ z)dz$$

$$\mathbf{Y}(\mathbf{r}_x)=h[\mathbf{m}(\mathbf{r}_x)\times(\mathbf{P}_n^{(+)}-\mathbf{P}_n^{(-)})]+\int_{-h}^{h}[\mathbf{m}(\mathbf{r}_x)\times\mathbf{F}(\mathbf{r}_x,\ z)]zdz$$

where $\mathbf{P}_n^{(\pm)}$ is the given vector of forces on the upper front surface S^+ and on the lower front surface S^- of the shell.

2. For the classical Timoshenko-type theory, according to the kinematic hypothesis, the displacement vector can be written in the form [4]

$$\mathbf{u}(\mathbf{R})=\mathbf{w}^{(0)}(\mathbf{r}_x)+\frac{z}{h}\mathbf{w}^{(1)}(\mathbf{r}_x),\quad \mathbf{w}^{(0)}=\mathbf{v}(\mathbf{r}_x),\quad \mathbf{w}^{(1)}=h\gamma_\alpha\mathbf{r}^\alpha \tag{2.1}$$

where $\mathbf{v}(\mathbf{r}_x)$ is the displacement vector of elements of the middle surface of the shell and $\gamma(\mathbf{r}_x)=\gamma_\alpha\mathbf{r}^\alpha$ is a vector which defines the rotation of fibres normal to the middle surface before deformation.

According to the static hypothesis of this theory it follows from (1.3) that

$$P^{33}=(\mathbf{P}^3,\ \mathbf{R}^3)=\lambda(\mathbf{R}^k,\ \partial_{k,x}\mathbf{u})+2\mu(\mathbf{m}_x,\ \partial_{3,x}\mathbf{u})=0$$

and the vectors $\mathbf{P}^\alpha$, $\mathbf{P}^3$ become

$$\mathbf{P}^\alpha=\lambda'(\mathbf{R}^\beta,\ \partial_{\beta,x}\mathbf{u})\mathbf{R}^\alpha+\mu(\mathbf{R}^\alpha,\ \partial_{k,x}\mathbf{u})\mathbf{R}^k+\mu g^{\alpha\beta}\partial_{\beta,x}\mathbf{u},\quad \lambda'=\frac{2\lambda\mu}{\lambda+2\mu} \tag{2.2}$$

$$\mathbf{P}^3=\mu[(\mathbf{m}_x,\ \partial_{\beta,x}\mathbf{u})\mathbf{R}^\beta+(\mathbf{R}_\beta,\ \partial_{3,x}\mathbf{u})\mathbf{R}^\beta]$$

According to the usual assumptions about basis vectors and the first metric tensor of a thin or shallow shell, as well as formulae (2.1) and (2.2), the vector of forces $\mathbf{T}^\alpha$ and of moments $\mathbf{M}^\alpha$ in the classical Timoshenko-type theory, allowing for the expressions for shearing forces [4] can be represented in the form

$$\mathbf{T}^\alpha = 2h[\lambda'(\mathbf{r}^\beta,\ D_{\beta,x}\mathbf{w}^{(0)})\mathbf{r}^\alpha + \mu(\mathbf{r}^\alpha,\ D_{k,x}\mathbf{w}^{(0)})\mathbf{r}^k + \mu a^{\alpha\beta} D_{\beta,x}\mathbf{w}^{(0)}] +$$
$$+2h\mu(k'-1)[(\mathbf{m}_x,\ D_{\beta,x}\mathbf{w}^{(0)})a^{\alpha\beta} + (\mathbf{r}^\alpha,\ D_{3,x}\mathbf{w}^{(0)})]\mathbf{m}_x \tag{2.3}$$
$$\mathbf{M}^\alpha = \frac{2h^2}{3}[\mathbf{m}_x \times \mathbf{t}^\alpha]$$
$$\mathbf{t}^\alpha = \lambda'(\mathbf{r}^\beta,\ D_{\beta,x}\mathbf{w}^{(1)})\mathbf{r}^\alpha + \mu(\mathbf{r}^\alpha,\ D_{k,x}\mathbf{w}^{(1)})\mathbf{r}^k + \mu a^{\alpha\beta} D_{\beta,x}\mathbf{w}^{(1)}$$
$$D_{k,x}\mathbf{w}^{(m)} = \frac{2m+1}{2h}\int_{-h}^{h} P_m\left(\frac{z}{h}\right)\partial_{k,x}\mathbf{u}(\mathbf{R})dz =$$
$$= \begin{cases} \partial_{\beta,x}\mathbf{w}^{(m)}, & k=\beta \\ \dfrac{2m+1}{2h}[\mathbf{u}^{(+)} - (-1)^m\mathbf{u}^{(-)} - 2\delta_{1m}\mathbf{w}^{(0)}], & k=3 \end{cases} \quad (m=0,\ 1)$$

where k is the shear factor, $\mathbf{u}^{(\pm)} = \mathbf{u}(\mathbf{r}_x \pm h\mathbf{m}_x)$, δ_{1m} is the Kronecker delta and $P_m(z/h)$ is a Legendre polynomial of order m.

For the vector $\mathbf{T}^3$ we have the formula

$$\mathbf{T}^3 = 2h\mu k'\left[(\mathbf{m}_x,\ \partial_{\beta,x}\mathbf{w}^{(0)})\mathbf{r}^\beta + \left(\mathbf{r}_\beta,\ \frac{1}{h}\mathbf{w}^{(1)}\right)\mathbf{r}^\beta\right] \tag{2.4}$$

We will introduce the differential operator $L\mathbf{u}$, which is defined as

$$L\mathbf{u}(\mathbf{r}_x,\ z) = \frac{1}{2h}\partial_{\alpha,x}(\sqrt{a}\mathbf{T}^\alpha) + \frac{3z}{2h^3}\{[\partial_{\alpha,x}(\sqrt{a}\mathbf{M}^\alpha)\times\mathbf{m}_x] + \sqrt{a}[[\mathbf{r}_\alpha \times \mathbf{T}^\alpha]\times\mathbf{m}_x]\} \tag{2.5}$$

and also by formulae (2.1) and (2.3), and is an approximation of the Lamé operator in the classical Timoshenko-type theory. Then for the vector constructed using the Kelvin displacement vector $\mathbf{U}_{(i)}$ (U_i^p is the Kelvin displacement tensor [5])

$$\mathbf{W}_{(i)}(\mathbf{r}_x,\ \Lambda) = \mathbf{U}_{(i)}^{(0)}(\mathbf{r}_x,\ \Lambda) + \frac{z}{h}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x,\ \Lambda) \tag{2.6}$$
$$\mathbf{U}_{(i)}^{(k)}(\mathbf{r}_x,\ \Lambda) = \frac{2k+1}{2h}\int_{-h}^{h} P_k\left(\frac{z}{h}\right)\mathbf{U}_{(i)}(\mathbf{R},\ \Lambda)dz,\quad k=0,\ 1$$
$$\mathbf{U}_{(i)} = U_i^p\mathbf{e}_p$$

we have the relation

$$L\mathbf{W}_{(i)} \equiv \frac{1}{2h}\partial_{\alpha,x}(\sqrt{a}\mathbf{T}_{(i)}^\alpha) + \frac{3z}{2h^3}\{[\partial_{\alpha,x}(\sqrt{a}\mathbf{M}_{(i)}^\alpha)\times\mathbf{m}_x + \sqrt{a}[[\mathbf{r}_\alpha\times\mathbf{T}_{(i)}^\alpha]\times\mathbf{m}_x]\} =$$
$$= -\sqrt{a}\left\{\frac{1}{2h}[\mathbf{T}_{(i)}^{(+)3} - \mathbf{T}_{(i)}^{(-)3} + \mathbf{e}_i\delta(\mathbf{r}_x - \mathbf{r}_\eta)] +\right.$$
$$\left.+\frac{3z}{2h^3}\{[(h[\mathbf{m}_x\times(\mathbf{T}_{(i)}^{(+)3} + \mathbf{T}_{(i)}^{(-)3})] + \xi\delta(\mathbf{r}_x - \mathbf{r}_\eta)[\mathbf{m}_x\times\mathbf{e}_i])\times\mathbf{m}_x]\}\right\} -$$
$$-\left\{\frac{1}{2h}\partial_{\alpha,x}(\sqrt{a}\mathbf{P}_{T(i)}^\alpha) + \frac{3z}{2h^3}[(\partial_{\alpha,x}(\sqrt{a}\mathbf{P}_{M(i)}^\alpha) + \sqrt{a}[\mathbf{r}_\alpha\times\mathbf{P}_{T(i)}^\alpha])\times\mathbf{m}_x]\right\},\quad \mathbf{r}_\eta\in S,\ |\xi|<h \tag{2.7}$$

Here

$$\mathbf{T}_{(i)}^\alpha(\mathbf{r}_x,\ \Lambda) = 2h[\lambda'(\mathbf{r}^\beta,\ D_{\beta,x}\mathbf{U}_{(i)}^{(0)})\mathbf{r}^\alpha + \mu(\mathbf{r}^\alpha,\ D_{k,x}\mathbf{U}_{(i)}^{(0)})\mathbf{r}^k +$$
$$+\mu a^{\alpha\beta}D_{\beta,x}\mathbf{U}_{(i)}^{(0)}] + 2h\mu(k'-1)[(\mathbf{m}_x,\ D_{\beta,x}\mathbf{U}_{(i)}^{(0)})a^{\alpha\beta} +$$

$$+(\mathbf{r}^\alpha,\ D_{3,x}\mathbf{U}^{(0)}_{(i)})]\mathbf{m}_x,\quad \mathbf{M}^\alpha_{(i)}(\mathbf{r}_x,\ \Lambda)=\frac{2h^2}{3}[\mathbf{m}_x\times\mathbf{t}^\alpha_{(i)}]$$

$$\mathbf{t}^\alpha_{(i)}(\mathbf{r}_x,\ \Lambda)=\lambda'(\mathbf{r}^\beta,\ D_{\beta,x}\mathbf{U}^{(1)}_{(i)})\mathbf{r}^\alpha+\mu(\mathbf{r}^\alpha,\ D_{k,x}\mathbf{U}^{(1)}_{(i)})\mathbf{r}^k+\mu a^{\alpha\beta}D_{\beta,x}\mathbf{U}^{(1)}_{(i)}$$

$$\mathbf{T}^{(\pm)3}_{(i)}=\mathbf{T}^3_{(i)}(\mathbf{r}_x\pm h\mathbf{m}(\mathbf{r}_x))$$

$$\mathbf{T}^3_{(i)}=\lambda(\mathbf{R}^k,\ \partial_{k,x}\mathbf{U}_{(i)})\mathbf{m}+\mu(\mathbf{m},\ \partial_{k,x}\mathbf{U}_{(i)})\mathbf{R}^k+\mu\partial_{3,x}\mathbf{U}_{(i)} \tag{2.8}$$

$$\mathbf{P}^\alpha_{T(i)}=\frac{\lambda 2h}{\lambda+2\mu}(\mathbf{T}^{(0)3}_{(i)},\ \mathbf{m}_x)\mathbf{r}^\alpha-2h\mu(k'-1)[(\mathbf{m}_x,\ D_{\beta,x}\mathbf{U}^{(0)}_{(i)})a^{\alpha\beta}+(\mathbf{r}^\alpha,\ D_{3,x}\mathbf{U}^{(0)}_{(i)})]\mathbf{m}_x$$

$$\mathbf{P}^\alpha_{M(i)}=\frac{2h^2}{3}\frac{\lambda}{\lambda+2\mu}(\mathbf{T}^{(1)3}_{(i)},\ \mathbf{m}_x)[\mathbf{m}_x\times\mathbf{r}^\alpha]$$

$$(\mathbf{T}^{(k)3}_{(i)}(\mathbf{r}_x,\ \Lambda),\ \mathbf{m}_x)=\lambda(\mathbf{r}^\beta,\ D_{\beta,x}\mathbf{U}^{(k)}_{(i)})+(\lambda+2\mu)(\mathbf{m}_x,\ D_{3,x}\mathbf{U}^{(k)}_{(i)}),\quad k=0,\ 1$$

$\delta(\mathbf{r}_x-\mathbf{r}_\eta)$ is the Dirac delta-function, $\mathbf{e}_i$ is the unit vector of a Cartesian system of coordinates defining the direction of operation of a point force in an infinite elastic medium and applied at the point with radius vector $\Lambda=\mathbf{r}_\eta+\xi\mathbf{m}_\eta$, $\mathbf{r}_\eta\equiv\mathbf{r}(\eta^1,\ \eta^2)$ is the radius vector of a point of the middle surface of a shell with Gaussian coordinates $(\eta^1,\ \eta^2)$, and ξ is the normal coordinate, taken along the normal $\mathbf{m}_\eta\equiv\mathbf{m}(\mathbf{r}\eta)$; by enclosing the subscript i in the parentheses we emphasize that the Kelvin vector $\mathbf{U}_{(i)}$ corresponds to the unit force $\mathbf{e}_i$ in a Cartesian system of coordinates, and the equation for this vector can be written in a curvilinear system of coordinates.

Suppose that the vector (2.1) is a solution of system (1.4), (1.5). Then we transform the integral

$$I=\int_{-h}^{h}dz\iint_S[(L\mathbf{u},\ \mathbf{W}_{(i)})-(L\mathbf{W}_{(i)},\ \mathbf{u})]dx^1dx^2=$$

$$=\iint_S\Big\{[(\partial_{\alpha,x}(\sqrt{a}\mathbf{T}^\alpha),\ \mathbf{U}^{(0)}_{(i)})-(\partial_{\alpha,x}(\sqrt{a}\mathbf{T}^\alpha_{(i)}),\ \mathbf{w}^{(0)})]+$$

$$+\frac{1}{h}[([\partial_{\alpha,x}(\sqrt{a}\mathbf{M}^\alpha)\times\mathbf{m}_x],\ \mathbf{U}^{(1)}_{(i)})-([\partial_{a,x}(\sqrt{a}\mathbf{M}^\alpha_{(i)})\times\mathbf{m}_x],\ \mathbf{w}^{(1)})]+$$

$$+\sqrt{a}\frac{1}{h}[(\mathbf{U}^{(1)}_{(i)},\ [[\mathbf{r}_\alpha\times\mathbf{T}^\alpha]\times\mathbf{m}_x])-(\mathbf{w}^{(1)},\ [[\mathbf{r}_\alpha\times\mathbf{T}^\alpha_{(i)}]\times\mathbf{m}_x])]\Big\}dx^1dx^2$$

which, according to (1.4), (1.5), (2.1) and (2.5)–(2.7), is equal to

$$I=\iint_S\sqrt{a}\Big\{-(\mathbf{X},\ \mathbf{U}^{(0)}_{(i)})-(\mathbf{U}^{(1)}_{(i)},\ [\mathbf{Y}\times\mathbf{m}_x])\frac{1}{h}+(\mathbf{T}^{(+)3}_{(i)}-\mathbf{T}^{(-)3}_{(i)},\ \mathbf{w}^{(0)})+$$

$$+(\mathbf{w}^{(0)},\ \mathbf{e}_i)\delta(\mathbf{r}_x-\mathbf{r}_\eta)+(\mathbf{T}^{(+)3}_{(i)}+\mathbf{T}^{(-)3}_{(i)},\ \mathbf{w}^{(1)})+\frac{\xi}{h}(\mathbf{w}^{(1)},\ \mathbf{e}_i)\delta(\mathbf{r}_x-\mathbf{r}_\eta)+$$

$$+\frac{1}{\sqrt{a}}(\partial_{\alpha,x}(\sqrt{a}\mathbf{P}^\alpha_{T(i)}),\ \mathbf{w}^{(0)})+$$

$$+\frac{1}{h}\left\{\left(\left[\frac{1}{\sqrt{a}}(\partial_{\alpha,x}(\sqrt{a}\mathbf{P}^\alpha_{M(i)})\times\mathbf{m}_x\right],\ \mathbf{w}^{(1)}\right)+([[\mathbf{r}_\alpha\times\mathbf{P}^\alpha_{T(i)}]\times\mathbf{m}_x],\ \mathbf{w}^{(1)})\right\}\Big\}dx^1dx^2 \tag{2.9}$$

to the form (n^Σ_α are the components of the unit outward normal to the surface Σ and ds_t is an element of the arc of the contour Γ)

$$I=\int_\Gamma\Big\{(\mathbf{T}^\alpha n^\Sigma_\alpha,\ \mathbf{U}^{(0)}_{(i)})-(\mathbf{T}^\alpha_{(i)}n^\Sigma_\alpha,\ \mathbf{w}^{(0)})+$$

$$+\frac{1}{h}[(\mathbf{M}^\alpha n^\Sigma_\alpha,\ [\mathbf{m}_x\times\mathbf{U}^{(1)}_{(i)}])-(\mathbf{M}^\alpha_{(i)}n^\Sigma_\alpha,\ [\mathbf{m}_x\times\mathbf{w}^{(1)}])]\Big\}ds_t-$$

$$-\iint_S\sqrt{a}\Big[(\mathbf{T}^\alpha,\ \partial_{\alpha,x}\mathbf{U}^{(0)}_{(i)})-(\mathbf{T}^\alpha_{(i)},\ \partial_{\alpha,x}\mathbf{w}^{(0)})+$$

$$+\frac{1}{h}[(\mathbf{M}^{\alpha},\ \partial_{\alpha,x}[\mathbf{m}_x\times\mathbf{U}^{(1)}_{(i)}])-(\mathbf{M}^{\alpha}_{(i)},\ \partial_{\alpha,x}[\mathbf{m}_x\times\mathbf{w}^{(1)}])\Big]+$$

$$+(\mathbf{T}^{\alpha},\ \mathbf{m}_x)\left(\frac{1}{h}\mathbf{U}^{(1)}_{(i)},\ \mathbf{r}_{\alpha}\right)-(\mathbf{T}^{\alpha}_{(i)},\ \mathbf{m}_x)\left(\frac{1}{h}\mathbf{w}^{(1)},\ \mathbf{r}_{\alpha}\right)\Bigg]dx^1dx^2 \qquad (2.10)$$

Using the relations

$$(\mathbf{T}^{\alpha},\ \partial_{\alpha,x}\mathbf{U}^{(0)}_{(i)})-(\mathbf{T}^{\alpha}_{(i)},\ \partial_{\alpha,x}\mathbf{w}^{(0)})-(\mathbf{T}^{\alpha}_{(i)},\ \mathbf{m}_x)\left(\frac{1}{h}\mathbf{w}^{(1)},\ \mathbf{r}_{\alpha}\right)+$$

$$+(\mathbf{T}^{\alpha},\ \mathbf{m}_x)\left(\frac{1}{h}\mathbf{U}^{(1)}_{(i)},\ \mathbf{r}_{\alpha}\right)=-\frac{1}{h}\left(\mathbf{T}^3,\ \frac{1}{2}(\mathbf{U}^{(+)}_{(i)}-\mathbf{U}^{(-)}_{(i)})-\mathbf{U}^{(1)}_{(i)}\right)$$

$$\frac{1}{h}\Big[\Big(\mathbf{M}^{\alpha},\partial_{\alpha,x}\big[\mathbf{m}_x\times\mathbf{U}^{(1)}_{(i)}\big]\Big)-\Big(\mathbf{M}^{\alpha}_{(i)},\partial_{\alpha,x}\big[\mathbf{m}_x\times\mathbf{w}^{(1)}\big]\Big)\Big]=-\Big(\mathbf{m}_x,\mathbf{U}^{(1)}_{(i)}\Big)\Big(\partial_{\alpha,x}\mathbf{m}_x,\frac{2h}{3}\mathbf{t}^{\alpha}\Big)$$

$$\iint_S\Big[\Big(\partial_{\alpha,x}\big(\sqrt{a}\mathbf{P}^{\alpha}_{T(i)}\big),\mathbf{w}^{(0)}\Big)+\frac{1}{h}\Big([\partial_{\alpha,x}(\sqrt{a}\mathbf{P}^{\alpha}_{M(i)})\times\mathbf{m}_x],\mathbf{w}^{(1)}\Big)\Big]\ dx^1dx^2=$$

$$=\int_{\Gamma}[(\mathbf{P}^{\alpha}_{T(i)}n^{\Sigma}_{\alpha},\mathbf{w}^{(0)})+\frac{1}{h}(\mathbf{P}^{\alpha}_{M(i)}n^{\Sigma}_{\alpha},[\mathbf{m}_x\times\mathbf{w}^{(1)}])]ds_t-$$

$$-\iint_S\sqrt{a}[(\mathbf{P}^{\alpha}_{T(i)},\partial_{\alpha,x}\mathbf{w}^{(0)})+\frac{1}{h}(\mathbf{P}^{\alpha}_{M(i)},\partial_{\alpha,x}[\mathbf{m}_x\times\mathbf{w}^{(1)}])]dx^1dx^2$$

$$\frac{1}{h}(\mathbf{P}^{\alpha}_{M(i)},\partial_{\alpha,x}[\mathbf{m}_x\times\mathbf{w}^{(1)}])=\frac{2h\lambda}{3(\lambda+2\mu)}(\mathbf{T}^{(1)3}_{(i)},\mathbf{m}_x)(\mathbf{r}^{\alpha},\partial_{\alpha,x}\mathbf{w}^{(1)})$$

$$(\mathbf{w}^{(1)},[[\mathbf{r}_{\alpha}\times\mathbf{P}^{\alpha}_{T(i)}]\times\mathbf{m}_x])=-(\mathbf{w}^{(1)},\mathbf{r}_{\alpha})(\mathbf{m}_x,\mathbf{P}^{\alpha}_{T(i)})$$

from (2.9) and (2.10) we obtain the integral representation of the displacement vector of elements of the shell $\mathbf{u}(\mathbf{r}_x, z) = \mathbf{w}^{(0)}(\mathbf{r}_x) + (z/h)\mathbf{w}^{(1)}(\mathbf{r}_x)$ in the form

$$(\mathbf{u}(\mathbf{r}_{\eta},\xi),\mathbf{e}_i)=\int_{\Gamma}\{(\mathbf{T}^{\Sigma}_n(\mathbf{r}_{x,\Gamma}),\mathbf{U}^{(0)}_{(i)}(\mathbf{r}_{x,\Gamma},\Lambda))-$$

$$-(\mathbf{T}^{\Sigma}_{n(i)}(\mathbf{r}_{x,\Gamma},\Lambda),\mathbf{w}^{(0)}(\mathbf{r}_{x,\Gamma}))+(\mathbf{M}^{\Sigma}_n(\mathbf{r}_{x,\Gamma}),[\mathbf{m}_x\times\frac{1}{h}\mathbf{U}^{(1)}_{(i)}(\mathbf{r}_{x,\Gamma},\Lambda)])-$$

$$-(\mathbf{M}^{\Sigma}_{n(i)}(\mathbf{r}_{x,\Gamma},\Lambda),[\mathbf{m}_x\times\frac{1}{h}\mathbf{w}^{(1)}(\mathbf{r}_{x,\Gamma})])\}ds_t+$$

$$+\iint_S\{(\mathbf{X}(\mathbf{r}_x),\mathbf{U}^{(0)}_{(i)}(\mathbf{r}_x,\Lambda))+([\mathbf{Y}(\mathbf{r}_x)\times\mathbf{m}_x],\frac{1}{h}\mathbf{U}^{(1)}_{(i)}(\mathbf{r}_x,\Lambda))-$$

$$-(\mathbf{T}^{(+)3}_{(i)}-\mathbf{T}^{(-)3}_{(i)},\mathbf{w}^{(0)}(\mathbf{r}_x))-(\mathbf{T}^{(+)3}_{(i)}+\mathbf{T}^{(-)3}_{(i)},\mathbf{w}^{(1)}(\mathbf{r}_x))+$$

$$+\frac{1}{h}(\mathbf{T}^3(\mathbf{r}_x),\frac{1}{2}(\mathbf{U}^{(+)}_{(i)}-\mathbf{U}^{(-)}_{(i)})-\mathbf{U}^{(1)}_{(i)}(\mathbf{r}_x,\Lambda))+$$

$$+(\mathbf{m}_x,\mathbf{U}^{(1)}_{(i)}(\mathbf{r}_x,\Lambda))(\partial_{\alpha,x}\mathbf{m}_x,\frac{2h}{3}\mathbf{t}^{\alpha}(\mathbf{r}_x))+(\mathbf{P}^{\alpha}_{T(i)}(\mathbf{r}_x,\Lambda),\partial_{\alpha,x}\mathbf{w}^{(0)}(\mathbf{r}_x))+$$

$$+\frac{2h\lambda}{3(\lambda+2\mu)}(\mathbf{T}^{(1)3}_{(i)}(\mathbf{r}_x,\Lambda),\mathbf{m}_x)(\mathbf{r}^{\alpha},\partial_{\alpha,x}\mathbf{w}^{(1)}(\mathbf{r}_x))+$$

$$+(\frac{1}{h}\mathbf{w}^{(1)}(\mathbf{r}_x),\mathbf{r}_{\alpha})(\mathbf{m}_x,\mathbf{P}^{\alpha}_{T(i)}(\mathbf{r}_x,\Lambda))\}dS_{r_x},\mathbf{r}_{\eta}\in S,\ \ |\xi|<h \qquad (2.11)$$

Here

$$\mathbf{T}^{\Sigma}_n=\mathbf{T}^{\alpha}n^{\Sigma}_{\alpha},\ \ \mathbf{M}^{\Sigma}_n=\mathbf{M}^{\alpha}n^{\Sigma}_{\alpha},\ \ \mathbf{T}^{\Sigma}_{n(i)}=2h\mathbf{P}^{(0)\Sigma}_{(i)}=(\mathbf{T}^{\alpha}_{(i)}+\mathbf{P}^{\alpha}_{T(i)})n^{\Sigma}_{\alpha}$$

$$\mathbf{M}^{\Sigma}_{n(i)}=\frac{2h^2}{3}[\mathbf{m}_x\times\mathbf{P}^{(1)\Sigma}_{(i)}]=(\mathbf{M}^{\alpha}_{(i)}+\mathbf{P}^{\alpha}_{M(i)})n^{\Sigma}_{\alpha}$$

$P_{(i)}^{(k)\Sigma}$ is the kth moment of the Kelvin stress vector [5] on the side surface of the shell.

3. For any internal point of the shell with radius vector Λ let the vectors $\mathbf{w}^{(0)}$, $\mathbf{w}^{(1)}$ satisfy vector relation (2.11). We will determine the error by which the vector $\mathbf{u}(\mathbf{r}_\eta, \xi)$ satisfies the system of equations (1.4) and (1.5). To do so, we introduce the operators

$$L_\eta^{(0)}\mathbf{v}(\Lambda) \equiv \partial_{\alpha,\eta}(\sqrt{a(\mathbf{r}_\eta)}(\mathbf{T}^\alpha(\mathbf{r}_\eta) + \mathbf{P}_T^\alpha(\mathbf{r}_\eta))) + \sqrt{a(\mathbf{r}_\eta)}(\mathbf{P}^{(+)3}(\mathbf{v}) - \mathbf{P}^{(-)3}(\mathbf{v})) \tag{3.1}$$

$$L_\eta^{(1)}\mathbf{v}(\Lambda) \equiv \partial_{\alpha,\eta}(\sqrt{a(\mathbf{r}_\eta)}(\mathbf{M}^\alpha(\mathbf{r}_\eta) + \mathbf{P}_M^\alpha(\mathbf{r}_\eta))) +$$

$$+\sqrt{a(\mathbf{r}_\eta)}([\mathbf{r}_\alpha \times (\mathbf{T}^\alpha(\mathbf{r}_\eta) + \mathbf{P}_T^\alpha(\mathbf{r}_\eta))] + h[\mathbf{m}_\eta \times (\mathbf{P}^{(+)3}(\mathbf{v}) + \mathbf{P}^{(-)3}(\mathbf{v}))]$$

$$\mathbf{P}_T^\alpha = \frac{2h\lambda}{\lambda + 2\mu}(\mathbf{T}^{(0)3}(\mathbf{v}), \mathbf{m}_\eta)\mathbf{r}^\alpha - 2h\mu(k' - 1)[(\mathbf{m}_\eta, D_{\beta,\eta}\mathbf{v}^{(0)})a^{\alpha\beta} + (\mathbf{r}^\alpha, D_{3,\eta}\mathbf{v}^{(0)})]\mathbf{m}_\eta$$

$$\mathbf{P}_M^\alpha = \frac{2h^2}{3}\frac{\lambda}{\lambda + 2\mu}(\mathbf{T}^{(1)3}(\mathbf{v}), \mathbf{m}_\eta)[\mathbf{m}_\eta \times \mathbf{r}^\alpha]$$

$$(\mathbf{T}^{(k)3}(\mathbf{v}), \mathbf{m}_\eta) = \lambda(\mathbf{r}^\beta, D_{\beta,\eta}\mathbf{v}^{(k)}(\mathbf{r}_\eta)) + (\lambda + 2\mu)(\mathbf{m}_\eta, D_{3,\eta}\mathbf{v}^{(k)}(\mathbf{r}_\eta))$$

$$\mathbf{v}^{(k)}(\mathbf{r}_\eta) = \frac{2k+1}{2h}\int_{-h}^{h} P_k\left(\frac{\xi}{h}\right)\mathbf{v}(\Lambda)d\xi, \quad k = 0, 1$$

$$\mathbf{P}^3(\mathbf{v}) = \lambda(\mathbf{r}^k, \partial_{k,\eta}\mathbf{v})\mathbf{m}_\eta + \mu(\mathbf{m}_\eta, \partial_{k,\eta}\mathbf{v})\mathbf{r}^k + \mu\partial_{3,\eta}\mathbf{v}$$

$$\mathbf{P}^{(\pm)3} = \mathbf{P}^3(\mathbf{r}_h \pm h\mathbf{m}_\eta)$$

The Kelvin vectors, their moments and derivatives satisfy the equations

$$L_\eta^{(0)}\mathbf{U}_{(i)}^{(0)}(\mathbf{r}_x, \Lambda) = \mathbf{l}_{0(i)}(\mathbf{r}_x, \mathbf{r}_\eta), \quad L_\eta^{(0)}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x, \Lambda) = L_\eta^{(1)}\mathbf{U}_{(i)}^{(0)}(\mathbf{r}_x, \Lambda) = 0$$

$$L_\eta^{(1)}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x, \Lambda) = \mathbf{l}_{1(i)}(\mathbf{r}_x, \mathbf{r}_\eta), \quad L_\eta^{(k)}\mathbf{U}_{(i)}^{(m)}(\mathbf{r}_{x,\Gamma}, \Lambda) = 0$$

$$L_\eta^{(k)}\mathbf{P}_{(i)}^{(m)\Sigma}(\mathbf{r}_{x,\Gamma}, \Lambda) = L_\eta^{(k)}\mathbf{U}_{(i)}^{(\pm)} = L_\eta^{(k)}\mathbf{T}_{(i)}^{(\pm)3} = 0, \quad m, k = 0, 1$$

$$L_\eta^{(0)}\mathbf{P}_{T(i)}^\alpha(\mathbf{r}_x, \Lambda) = \frac{\lambda 2h}{\lambda + 2\mu}(\mathbf{T}_{\delta(i)}^{(0)3}, \mathbf{m}_x)\mathbf{r}^\alpha - 2h\mu(k' - 1)(\mathbf{m}_x, D_{\beta,x}\mathbf{l}_{0(i)}(\mathbf{r}_x, \mathbf{r}_\eta))a^{\alpha\beta}\mathbf{m}_x$$

$$L_\eta^{(1)}\mathbf{P}_{T(i)}^\alpha(\mathbf{r}_x, \Lambda) = 0$$

$$L_\eta^{(0)}\mathbf{P}_{M(i)}^\alpha(\mathbf{r}_x, \Lambda) = -2\lambda(\mathbf{m}_x, \mathbf{l}_{0(i)}(\mathbf{r}_x, \mathbf{r}_\eta))[\mathbf{m}_x \times \mathbf{r}^\alpha]$$

$$L_\eta^{(1)}\mathbf{P}_{M(i)}^\alpha(\mathbf{r}_x, \Lambda) = \frac{2h^2}{3}\frac{\lambda}{\lambda + 2\mu}(\mathbf{T}_{\delta(i)}^{(1)3}, \mathbf{m}_x)[\mathbf{m}_x \times \mathbf{r}^\alpha]$$

$$(\mathbf{T}_{\delta(i)}^{(k)3}, \mathbf{m}_x) = \lambda(\mathbf{r}^\beta, D_{\beta,x}\mathbf{l}_{k(i)}(\mathbf{r}_x, \mathbf{r}_\eta)), \quad k = 0, 1$$

$$\mathbf{l}_{0(i)}(\mathbf{r}_x, \mathbf{r}_\eta) = -\sqrt{a(\mathbf{r}_\eta)}\delta(\mathbf{r}_x - \mathbf{r}_\eta)\mathbf{e}_i$$

$$\mathbf{l}_{1(i)}(\mathbf{r}_x, \mathbf{r}_\eta) = \sqrt{a(\mathbf{r}_\eta)}\delta(\mathbf{r}_x - \mathbf{r}_\eta)[\mathbf{m}_\eta \times \mathbf{e}_i]h$$

From these equations and formulae (2.4) and (3.1), applying operators $L_\eta^{(k)}\mathbf{u}(\Lambda)$ to the right- and left-hand sides of relation (2.11), we obtain the system of vector equations

$$\begin{gathered}\partial_{\alpha,\eta}(\sqrt{a(\mathbf{r}_\eta)}\mathbf{T}^\alpha(\mathbf{r}_\eta)) + \sqrt{a(\mathbf{r}_\eta)}(\mathbf{X}(\mathbf{r}_\eta) + 2\mu h b_\beta^\alpha \gamma_\alpha(\mathbf{r}_\eta)\mathbf{r}^\beta) = 0 \\ \partial_{\alpha,\eta}(\sqrt{a(\mathbf{r}_\eta)}\mathbf{M}^\alpha(\mathbf{r}_\eta)) + \sqrt{a(\mathbf{r}_\eta)}([\mathbf{r}_\alpha \times \mathbf{T}^\alpha(\mathbf{r}_\eta)] + \mathbf{Y}(\mathbf{r}_\eta)) = 0\end{gathered} \tag{3.2}$$

It follows from (3.2) that the vector $\mathbf{u}(\mathbf{r}_\eta, \xi)$ satisfies Eq. (1.4) with an error $2\sqrt{(a(\mathbf{r}_\eta))}\mu h b_\beta^\alpha \gamma_\alpha(\mathbf{r}_\eta, \xi)$. In the approximations of a thin or shallow shell, these terms in the equilibrium equations can be neglected.

Using integral representation (2.11) for internal points of the shell, we can calculate the vectors of forces and moments from the formulae

$$\mathbf{T}^{\alpha}(\mathbf{r}_{\eta}) = l_{\eta}^{T}\mathbf{u}(\mathbf{r}_{\eta},\xi), \quad \mathbf{M}^{\alpha}(\mathbf{r}_{\eta}) = l_{\eta}^{M}\mathbf{u}(\mathbf{r}_{\eta},\xi)$$

$$l_{\eta}^{T} \equiv \int_{-h}^{h}\{\tilde{l}_{\eta} + 2h\mu(k'-1)[(\mathbf{m}_{\eta},\partial_{\beta,\eta})a^{\alpha\beta} + (\mathbf{r}^{\alpha},\partial_{3,\eta})]\mathbf{m}_{\eta}\}d\xi$$

$$l_{\eta}^{M} \equiv \int_{-h}^{h}\{[\mathbf{m}_{\eta}\xi\times\tilde{l}_{\eta}]\}d\xi$$

$$\tilde{l}_{\eta} = \lambda'(\mathbf{r}^{\beta},\partial_{\beta,\eta})\mathbf{r}^{\alpha} + \mu(\mathbf{r}^{\alpha},\partial_{k,\eta})\mathbf{r}^{k} + \mu a^{\alpha\beta}\partial_{\beta,\eta}$$

$$l_{\eta}^{T(M)}\mathbf{u}(\mathbf{r}_{\eta},\xi) = \mathbf{e}_i\Big\{\int_{\Gamma}[(\mathbf{T}_n^{\Sigma}(\mathbf{r}_{x,\Gamma}), l_{\eta}^{T(M)}\mathbf{U}_{(i)}^{(0)}(\mathbf{r}_{x,\Gamma},\Lambda)) -$$

$$-(l_{\eta}^{T(M)}\mathbf{T}_{n(i)}^{\Sigma}(\mathbf{r}_{x,\Gamma},\Lambda),\mathbf{w}^{(0)}(\mathbf{r}_{x,\Gamma})) + (\mathbf{M}_n^{\Sigma}(\mathbf{r}_{x,\Gamma}),[\mathbf{m}_x\times\frac{1}{h}l_{\eta}^{T(M)}\mathbf{U}_i^{(1)}(\mathbf{r}_{x,\Gamma},\Lambda)]) -$$

$$-(l_{\eta}^{T(M)}\mathbf{M}_{n(i)}^{\Sigma}(\mathbf{r}_{x,\Gamma},\Lambda),\ [\mathbf{m}_x\times\frac{1}{h}\mathbf{w}^{(1)}(\mathbf{r}_{x,\Gamma})])\Big\}ds_t +$$

$$+\iint_{S}\{(\mathbf{X}(\mathbf{r}_x),\ l_{\eta}^{T(M)}\mathbf{U}_{(i)}^{(0)}(\mathbf{r}_x,\Lambda)) + ([\mathbf{Y}(\mathbf{r}_x)\times\mathbf{m}_x],\ \frac{1}{h}l_{\eta}^{T(M)}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x,\Lambda)) -$$

$$-(l_{\eta}^{T(M)}(\mathbf{T}_{(i)}^{(+)3} - \mathbf{T}_{(i)}^{(-)3}),\mathbf{w}^{(0)}(\mathbf{r}_x)) - (l_{\eta}^{T(M)}(\mathbf{T}_{(i)}^{(+)3} + \mathbf{T}_{(i)}^{(-)3}),\mathbf{w}^{(1)}(\mathbf{r}_x)) +$$

$$+\frac{1}{h}(\mathbf{T}^3(\mathbf{r}_x), l_{\eta}^{T(M)}\frac{1}{2}(\mathbf{U}_{(i)}^{(+)} - \mathbf{U}_{(i)}^{(-)}) - l_{\eta}^{T(M)}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x,\Lambda)) +$$

$$+(\mathbf{m}_x, l_{\eta}^{T(M)}\mathbf{U}_{(i)}^{(1)}(\mathbf{r}_x,\Lambda))(\partial_{\alpha,x}\mathbf{m}_x,\frac{2h}{3}\mathbf{t}^{\alpha}(\mathbf{r}_x)) +$$

$$+(l_{\eta}^{T(M)}\mathbf{P}_{T(i)}^{(\alpha)}(\mathbf{r}_x,\Lambda),\partial_{\alpha,x}\mathbf{w}^{(0)}(\mathbf{r}_x)) + \frac{2h\lambda}{3(\lambda+2\mu)}(l_{\eta}^{T(M)}\mathbf{T}_{(i)}^{(1)3}(\mathbf{r}_x,\Lambda),\mathbf{m}_x)(\mathbf{r}^{\alpha},\partial_{\alpha,x}\mathbf{w}^{(1)}(\mathbf{r}_x)) +$$

$$+(\frac{1}{h}\mathbf{w}^{(1)}(\mathbf{r}_x),\mathbf{r}_{\alpha})(\mathbf{m}_x, l_{\eta}^{T(M)}\mathbf{P}_{T(i)}^{\alpha}(\mathbf{r}_x,\Lambda))\}dS_{r_x}\},\quad \mathbf{r}_{\eta}\in S,\ |\xi|<h$$

4. On the basis of integral representation (2.11), a boundary integral equation can be constructed for the unknown vectors both on the contour and on the middle surface of the shell. To construct this equation, we need to use the limit properties of the right-hand side of Eq. (2.11) on the side Σ.

We transform the sum of the integrals in (2.11)

$$I_{(i)}(\Lambda) = \int_{\Gamma}[-(\mathbf{T}_{n(i)}^{\Sigma}(\mathbf{r}_{x,\Gamma},\Lambda),\mathbf{w}^{(0)}(\mathbf{r}_{x,\Gamma})) - (\mathbf{M}_{n(i)}^{\Sigma}(\mathbf{r}_{x,\Gamma},\Lambda),\ [\mathbf{m}_x\times\frac{1}{h}\mathbf{w}^{(1)}(\mathbf{r}_{x,\Gamma})])]ds_t +$$

$$+\iint_{S}[-(\mathbf{T}_{(i)}^{(+)3} - \mathbf{T}_{(i)}^{(-)3},\mathbf{w}^{(0)}(\mathbf{r}_x)) - (\mathbf{T}_{(i)}^{(+)3} + \mathbf{T}_{(i)}^{(-)3},\mathbf{w}^{(1)}(\mathbf{r}_x))]dS_{r_x}$$

using (2.1) to the form

$$I_{(i)}(\Lambda) = -\iint_{(S^+\cup S^-\cup\Sigma)}(\mathbf{P}_{n(i)}(\mathbf{R},\Lambda),\mathbf{u}(\mathbf{r}_x,z) - \mathbf{u}(\mathbf{r}_{\eta},\xi))dS - \iint_{(S^+\cup S^-\cup\Sigma)}(\mathbf{P}_{n(i)}(\mathbf{R},\Lambda),\mathbf{u}(\mathbf{r}_{\eta},\xi))dS \quad (4.1)$$

where $\mathbf{P}_{n(i)}$ is the Kelvin stress vector on the shell surface.

Suppose that the vector $\mathbf{u}(\mathbf{r}_x, z)$ on this surface satisfies the Hölder condition. Then the first integral of the right-hand side of (4.1) does not experience a discontinuity on crossing the point with radius vector $\Lambda_p = \mathbf{r}_{\eta}^{p} + \xi^{p}\mathbf{m}_{\eta}(\mathbf{r}_{\eta}^{p})$ on the side Σ, and the integral $I_{(i)}$ possesses the following limit properties [5]

$$I_{(i)}^{\pm}(\Lambda_p) = I_{(i)}^{\Sigma}(\Lambda_p) \pm \frac{1}{2}(\mathbf{u}(\mathbf{r}_{\eta}^{p},\xi^{p}),\mathbf{e}_i)$$

where $I^{\pm}_{(i)}(\Lambda_p)$ are limit values of the integral $I_{(i)}$ at the point with radius vector $\Lambda_p = \mathbf{r}^p_\eta + \xi^p \mathbf{m}_\eta(\mathbf{r}^p_\eta)$ from inside (the plus exponent) and outside (the minus exponent) the shell, respectively; $I^{\Sigma}_{(i)}(\Lambda_p)$ is the direct (singular) value of this integral on Σ. It can be shown that the other integrals on the right-hand side of (2.11) do not suffer a discontinuity on crossing Σ.

The integral equations for $\mathbf{w}^{(0)}$ and $\mathbf{w}^{(1)}$ can be picked out from (2.11) by using the expansion of this relation in terms of Legendre polynomials $P_k(\xi/h)$ $(k = 0.1)$ and the limit properties of the integral $I_{(i)}$ on the surface Σ.

The integral equations for $\mathbf{w}^{(0)}$ and $\mathbf{w}^{(1)}$ thus constructed are the basis for constructing a boundary-element method for determining the parameters of the stress–strain state of the shell. By subdividing the middle surface and contour of the shell into isoparametric [5] boundary elements with cubic interpolation of the geometric and mechanical variables, we can reduce the integral equations for $\mathbf{w}^{(0)}$ and $\mathbf{w}^{(1)}$ to an algebraic system of equations. The nodal vectors of displacements both on the contour of the shell will be the unknowns of this system. Once the nodal unknowns have been determined, the forces and moments at internal points of the shell can be determined using the above formulae.

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